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ESTIMATING SIMPLE CLOSED CONTOURS IN IMAGES

Bennett R. Groshong, PhD.
PO Box 3012 Duke University Medical Center Durham, NC 27710

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Abstract

A technique for estimating the boundary of objects that may be described by a simple closed contour is presented. The problem is posed as one of maximum a posteriori (MAP) optimization. The method employs an elliptical Fourier series to describe tbe contour. A scaled derivative-of-Gaussian description of the object boundary cross-section is employed, coupled with guided Gradient descent estimation of the contour coefficients. The technique is applied to x-ray images of the left ventricle, but is easily extensible to a wide variety of images of similar objects. It is shown that a few elliptical harmonics accurately model the ventricle outline. Accurate, robust estimation of the left ventricle outline from a single image is sbown for a set of images. The method is easily extended to otber imaging modalities such as PET, SPECT, MRI, ultrasound, CAT, and intravascular ultrasound images of two, three, or four dimensions, requiring only elliptical harmonic parameters for the additional spatial and/or temporal dimensions and a boundary model of appropriate dimension.

Introduction

There are many areas of medical imaging that require estimation of the boundary of an object which may be described by a simple closed contour. Current practice for a large number of applications is to hand-trace the desired boundary, which is less than desirable in terms of both time and accuracy [3, 7].

A technique for estimating the boundary of objects with simple closed contours is presented in this paper. It is composed of five parts: 1) An elliptical Fourier series description of the contour; 2) A scaled derivative-of-Gaussian description of the object boundary cross-section; 3) A maximum a posteriori (MAP) formulation of the optimization problem; The use of iterative gradient descent to perform the optimization; 4) A guiding constraint on the contour coefficients. The technique is applied to single view X-ray images of the human left ventricle, such as the one illustrated in Figure 1. It is shown that a few elliptical harmonics accurately model the ventricle outline. Accurate, robust estimation of the left ventricle outline from a single image is shown for a set of ventriculograms. The method is easily extended to other imaging modalities such as PET, SPECT, MRI, ultrasound, CAT, and intravascular ultrasound images of two, three, and four dimensions, requiring only elliptical harmonic parameters for the additional spatial and/or temporal dimensions and a boundary cross-section model of appropriate dimension. The optimization method used

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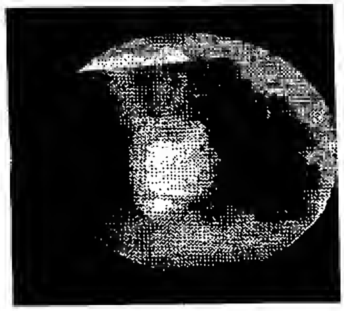

Figure 1: X-ray Ventriculogram.

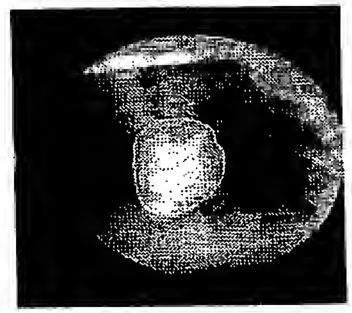

Figure 2: Optimized contour.

to determine the outline of the ventricle is described in section , with results for additional ventriculograms shown in section . A short discussion follows in section .

Method

There are many applications in which it is desirable to quickly, accurately estimate the size or outline of an object in an image with simple closed contour. For this application, the contour of interest is the boundary of the left ventricle in a ventriculogram, as in Figure 1. One method of describing object boundaries is an elliptic Fourier series, which may used to describe *any* closed contour to an arbitrary level of detail [4, 5]. The outline of the left ventricle is a simple, non-overlapping, smooth contour, as illustrated in Figure 3 A contonr such as that in Figure 3 may be written as a parameterized curve

$$C(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}. \tag{1}$$

If $C(t)$ is a closed contour, then $x(t), y(t)$ are periodic in t with period T, and may be described by an elliptical Fourier series [4]

$$x(t) = x_0 + \sum_{k=1}^{\infty} a_k \cos\frac{2k\pi t}{T} + b_k \sin\frac{2k\pi t}{T} \tag{2}$$

$$y(t) = y_0 + \sum_{k=1}^{\infty} c_k \cos\frac{2k\pi t}{T} + d_k \sin\frac{2k\pi t}{T} \tag{3}$$

The terms $x_0, y_0, a_k, b_k, c_k, d_k$ are defined as follows:

$$x_0 = \frac{1}{T}\int_{t=0}^{T} x(t)\,dt \tag{4}$$

$$y_0 = \frac{1}{T}\int_{t=0}^{T} y(t)\,dt \tag{5}$$

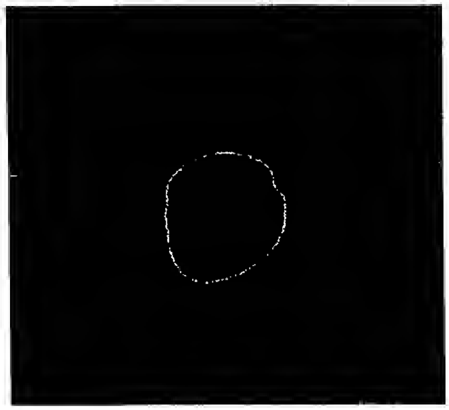

Figure 3: Left ventricle outline

$$a_k = \frac{2}{T}\int_{t=0}^{T} x(t)\cos\frac{2k\pi t}{T}\,dt \tag{6}$$

$$b_k = \frac{2}{T}\int_{t=0}^{T} x(t)\sin\frac{2k\pi t}{T}\,dt \tag{7}$$

$$c_k = \frac{2}{T}\int_{t=0}^{T} y(t)\cos\frac{2k\pi t}{T}\,dt \tag{8}$$

$$d_k = \frac{2}{T}\int_{t=0}^{T} y(t)\sin\frac{2k\pi t}{T}\,dt \tag{9}$$

Describing a contour as an elliptical Fourier series offers several attractive features for boundary description [8]: 1) Truncating the series at any harmonic N produces a closed curve; 2) The parameters are inherently geometrical in nature in that they describe ellipses; 3) Invariance to rotation and scale are easily achieved; 4) Partial derivatives may be calculated with respect to the parameterization variable and to the coefficients. Describing the object boundary with a truncated series is a desireable objective. As with Fourier analysis of time signals, limiting the number of terms smooths the signal. For the boundaries of interest in this case, a small number (4-8) of terms are required to accurately describe the object. Given a method of describing the ventricle outline, we next need a method of relating the outline to the image.

The ventricle is a light object on a dark background as can be seen in Figure 1. It is convenient to assume that the intensity profile as the object boundary is crossed in a direction normal to the boundary is a step change in intensity. Canny [1, 2] described an optimal method for the detection of a step change in intensity in the presence of noise, given the simultaneous objectives of maximizing the probability of detecting the edge, the accuracy with which the edge is located, and producing a single response to the edge. Furthermore, Canny showed that the first derivative of a Gaussian both closely approximates the optimal

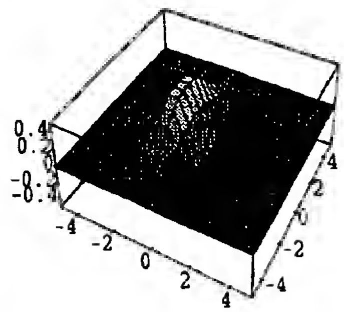

Figure 4: Derivative of a Gaussian

shape and may be efficiently computed. Experiments with edge detectors and templates have indicated that the first derivative of a Gaussian, described mathematically as

$$M_r = -\frac{(r)}{\sqrt{2\pi}\sigma^3} \exp\left\{-\frac{(r)^2}{2\sigma^2}\right\}, \tag{10}$$

where r is the normal distance to the contour. The function, shown in Figure 4, is an excellent choice for the detection of boundaries in images. The derivative of a Gaussian is used to detect and locate the edge of the objct in the presence of noise. The scale of the Gaussian derivative blurring function is matched to the pixel scale of the object at the boundary. Boundaries with cross-sectional shapes such as that of a line or ridge may be easily modeled by appropriate derivative-of-Gaussian functions. The elliptical harmonic contour is combined with the derivative-of-Gaussian edge function to produce a parameterized merit image $f(x(p), y(p), r)$, which, when convolved with the observed image g, will provide a global measure of the similarity between an instance of the contour and the object. This measure will pass through a well defined maximum when the parametric curve matches that of the underlying object boundary, as described by Canny [1]
Finding the contour that is somehow "best", or optimal, given the image at hand, may be phrased in a probabalistic manner as the maximum a posteriori probability (MAP) of the contour with respect to the image. This is the conditional probability of the parameterized contour given the image $P(p/g)$. Using Baye's rule, this may be written as

$$P(p/g) = \frac{P(g/p)P(p)}{P(g)}. \tag{11}$$

Evaluating and optimizing the posterior probability requires that some assumptions be made about the observed image and contour. If we assume that: The function f accurately models the image content, with the exception of noise; The noise is additive and Gaussian in nature; The noise is independent for each image location; The noise exhibits an identical

standard deviation for each image location. Then the conditional probability of the image, given the model function may be written as

$$P(g/p) = P(g/f(p)) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_n} \exp - \frac{(g_i - f_i)^2}{2\sigma_n^2} \tag{12}$$

for a set of points $\{i\}$ generated from the independent variables $\{p, r, t\}$ over the range of the merit function f.

The prior likelihood of an instance of the contour is described by a probability distribution on the elliptical contour parameters, simply modeled as an independent multivariate Gaussian

$$P(p) = \prod_j \frac{1}{\sqrt{2\pi}\sigma_j} \exp - \frac{(p_j - \mu_j)^2}{2\sigma_j^2}. \tag{13}$$

A maximum a posteriori (MAP) optimization problem is formulated by combining this prior probability with the conditional probability of the contour given the image. The posterior probability (eqn. 11) is simplified so that it may be optimized by gradient descent by taking the natural logarithm, giving

$$\ln P(p/g) = \ln P(g/p) + \ln P(p) - \ln P(g) \tag{14}$$

The probability of the observed image $P(g)$ is assumed to be constant with respect to the optimization, and is not considered further. Expanding the remaining terms of eqn. 14, we have

$$\ln P(p/g) = \ln P(f(p)/g) = \sum_i \left(\ln 1 - \ln \sqrt{2\pi}\sigma_n - \frac{(g_i - f_i)^2}{2\sigma_n^2}\right) + \tag{15}$$

$$\sum_j \left(\ln 1 - \ln \sqrt{2\pi}\sigma_j - \frac{(p_j - \mu_j)^2}{2\sigma_j^2}\right) \tag{16}$$

Gradient descent is performed on the log probability (eqn. 15) to determine the contour parameters giving the maximum posterior probability. The partial derivatives of the log probability with respect to the parameters are

$$\frac{\partial \ln P(p/g)}{\partial p_k} = \frac{\partial \ln P(g/p)}{\partial p_k} + \frac{\partial \ln P(p)}{\partial p_k}. \tag{17}$$

If we expand the terms, we arrive at

$$\frac{\partial \ln P(p/g)}{\partial p_k} = \sum_i \left(-\frac{(g_i - f_i)}{\sigma_n^2}\left(\frac{\partial g_i}{\partial p_k} - \frac{\partial f_i}{\partial p_k}\right)\right) - \left(\frac{(p_j - \mu_j)}{\sigma_j^2}\right). \tag{18}$$

The parameters are optimized by beginning at the prior mean of the parameters and traveling up the gradient of the log posterior probability by iteratively adding the gradient to the current parameter set

$$\hat{p}^{k+1} = \hat{p}^k - \alpha \frac{\partial P(\hat{p}^k/g)}{\partial \hat{p}} \tag{19}$$

to find a maximum.

The gradient descent is guided along a path of solutions by optimizing the parameters in increasing order. The gradient is limited to M terms, where M is varied from 0 –

standard deviation for each image location. Then the conditional probability of the image, given the model function may be written as

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to find a maximum.

The gradient descent is guided along a path of solutions by optimizing the parameters in increasing order. The gradient is limited to M terms, where M is varied from 0 –

N sequentially. This technique forces the gradient to optimize the terms describing the contour in order of decreasing confidence. The contour translation $\{x_0, y_0\}$ is determined first, then the ellipse $\{x_0, y_0, a_1, b_1, c_1, d_1\}$ approximating the object, then the higher terms in increasing order. The result is an optimization algorithm that, given a prior describing the type of object and a noisy image of an instance of the object, will reliably determine contours that agree with human intuition.

Results

The results given in this section illustrate the performance of the optimization algorithm described previously on two examples of digital subtraction X-ray ventriculograms.
The prior used in each case was a circle located in the center of the image, with a radius roughly approximating the ventricle size, and a large standard deviation on the parameters. This was a fairly naive assumption, as the ventricle at systoly and at diastoly is of a fairly consistent size and shape for adult humans. The excellent performance indicated for the algorithm, as illustrated in Figures 5, 6, 7, 8 using a naive prior leads us to believe that a more specific prior determined from many ventricle images would be able to withstand more noise and/or object artefacts in the image.

Discussion

This paper presents a solution to the problem of estimating object contours in images. Key features are the use of an elliptical Fourier series to represent the contour, the use of a boundary measure tolerant of noise, the use of prior knowledge of the contour, and guided optimization of the contour parameters. The work presented is similar in some areas to the methods discussed by Staib [8], but differs from in that it uses a boundary model incorporating knowledge of the object brightness contour, optimizes the posterior probability by gradient descent, and guides the optimization by working with the lower order parameters first. As mentioned in [8], a resolution pyramid strategy should offer advantages in both efficiency and performance. Extension to image sequences is straightforward, simply requiring description of the contour parameters in terms of a Fourier time series, as described in Schudy [6].

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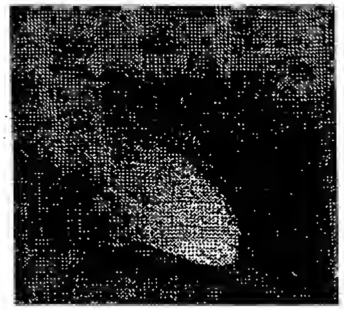

Figure 5: Ventriculogram at diastoly.

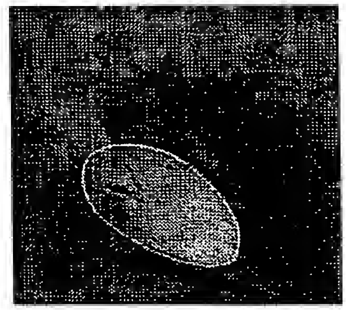

Figure 6: Optimized contour overlayed on diastolic image.

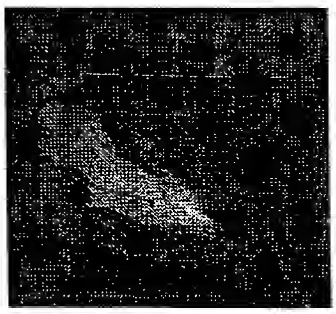

Figure 7: Ventriculogram at systoly.

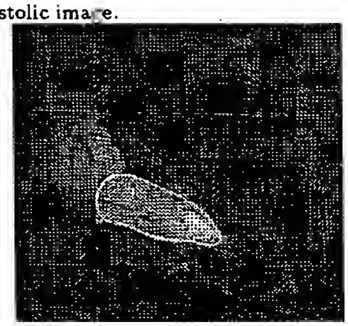

Figure 8: Optimized contour overlayed on systolic image.

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